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Unsteady-State Model for Continuous Moving-Bed Chromatography

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ABSTRACT

Mathematical equations have been formulated to predict the longitudinal concentration distribution in the case of instantaneous behavior of a continuous chromatographic system. Partial differential equations are initially obtained from mass balance on the differential element of column height. The formulated equations are solved by Laplace transformation and the contour integration method. Two equations are obtained, one for the column section above the feed point and the second for the column section below the feed. Both of these equations give the concentration distribution along the appropriate section of the column. The model predicts the instantaneous and steady-state behavior of the continuous moving-bed chromatographic system. The model can also be used as a tool for the design of new continuous chromatographic columns.

Key Words. Moving-bed chromatography; Unsteady-state separation; Mathematical model for continuous chromatography; Application of contour integration method; Design of continuous chromatographic columns

INTRODUCTION

Gas–liquid chromatography (GLC) is a powerful separating tool for both analytical and productive techniques. It was initially developed by James and Martin (1) and since then it has received considerable attention from scientists and engineers.

Tatlow and Evans (2) were the first to use a productive GLC column, and they were followed by Barker and Critcher (3) and Baker and Huntington (4). The first generation of continuous countercurrent adsorption processes, developed by Berg (5) and Kehde et al. (6), proved to be uneconomical due to difficulties associated with circulation of the solid adsorbent. Such problems were eliminated in the simulated moving-bed systems of Broughton et al. (7) and De Rosset et al. (8) by moving the feed and draw-off points to intervals through a fixed adsorbent bed which is divided into many sections. Such processes have been employed in industry for several hydrocarbon separations as well as for the separation of glucose and fructose in the production of high fructose syrup as presented by Neuzil and Jensen (9). Various other types of GLC columns have been developed so that columns greater than 5 m in diameter and production rates up to 2,273 ton/yr have been used by Ryan and Timmins (10).

Theoretical models for simulating GLC columns have been studied by several workers. Tiley (11) found that the efficiency of separation is dependent on the ratio of solvent to feed and on the column temperature. Yoon and Kunii (12) obtained an analytical solution for a moving system by assuming a linear isotherm and plug flow, and they compared the applicability of the obtained model with their experimental results.

Martin and Synge (13) were the first to realize the similarity between transfer processes and those which occur in chromatography, and they developed a GLC model theory. James and Martin (1) developed the concept of gas–liquid partition chromatography.

Several workers, among them Purnell (14) and Chreshire and Scot (15), extensively studied the effect of solid particle size, carrier gas velocity, column dimensions, and temperature on column performance and efficiency. Their work was carried out mostly on analytical columns and was expected to be equally applicable to the performance of large-scale and continuous columns. Ching (16), Barker et al. (17), and Hashimoto et al. [cited by Ruthven (18)] presented theoretical models based on plate calculations. On the other hand, Ching and Ruthven (19) simulated the countercurrent adsorption system to present concentration profiles of separating glucose and fructose under a wide range of operating conditions. They proved that calculations from a dispersed plug flow model, with the dispersion coefficient obtained from pulse chromatographic measurements, agreed well with experimental values.

Almost all mathematical expressions found in the literature, with the exception of those of Ching and Ruthven (19), are derived for steady-state conditions. The present work studies the possibility of formulating a theoretical model which can be utilized for unsteady- and steady-state separation of two solutes by continuous moving-bed chromatography columns.

MATHEMATICAL MODEL

A mathematical model to simulate the operation of continuous moving-bed chromatography can be obtained by visualizing the system shown in Fig. 1. It begins with a vertical cylindrical column. Spherical porous solid particles are introduced at its top and move downward at a constant superficial velocity (U_s) as a result of the gravitational effect. An inert gas is introduced from the column bottom, and it flows by countercurrent upward through the moving bed under isothermal and isobaric conditions. The binary feed mixture to be separated is introduced at a predetermined location along the column. One of the components of the mixture is preferentially carried upward by the inert gas. This component is usually referred to as the light component. The other component is preferentially captured by the nonvolatile liquid phase impregnated within the solid support and travels downward. The adsorbed solute is revaporized at the bottom of the column by the preheated inert gas stream (usually at a higher temperature).

In deriving a mathematical expression that can be used for predicting the concentration distribution within the column, the following assumptions can be used without affecting the separation processes:

(a) Phases flow within the column at a constant rate, and solid particles flow with a constant void fraction.
(b) Longitudinal dispersion in the vapor and solid phases is negligible in comparison with convective transport.
(c) Concentration distribution in the radial direction within the bed is neglected.
(d) Mass flux is proportional to the concentration difference between the phases.
(e) Isothermal and isobaric adsorption processes are used.
(f) Particle diameters are small in comparison with column dimensions, and each particle is uniformly covered with the gas mixture.
(g) A constant mass transfer coefficient is taken during adsorption. Take a mass balance on an element of thickness (dy) of the moving bed at a distance (y) from the top of the section under consideration, as shown in Fig. 2. Use the general mass transfer equation:

$$\text{Input} - \text{Output} = \text{Accumulation} - \text{Loss by sorption} \qquad (1)$$

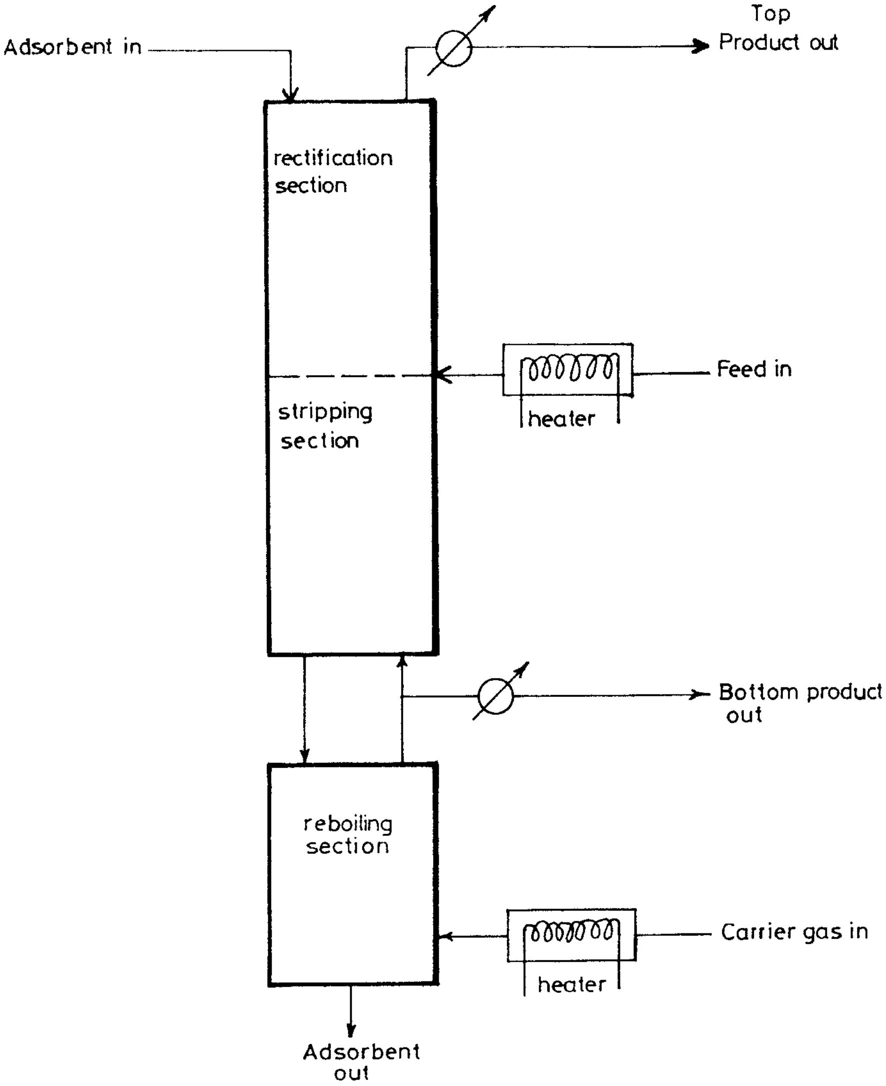

FIG. 1 Schematic diagram for separation process by continuous moving-bed chromatography.

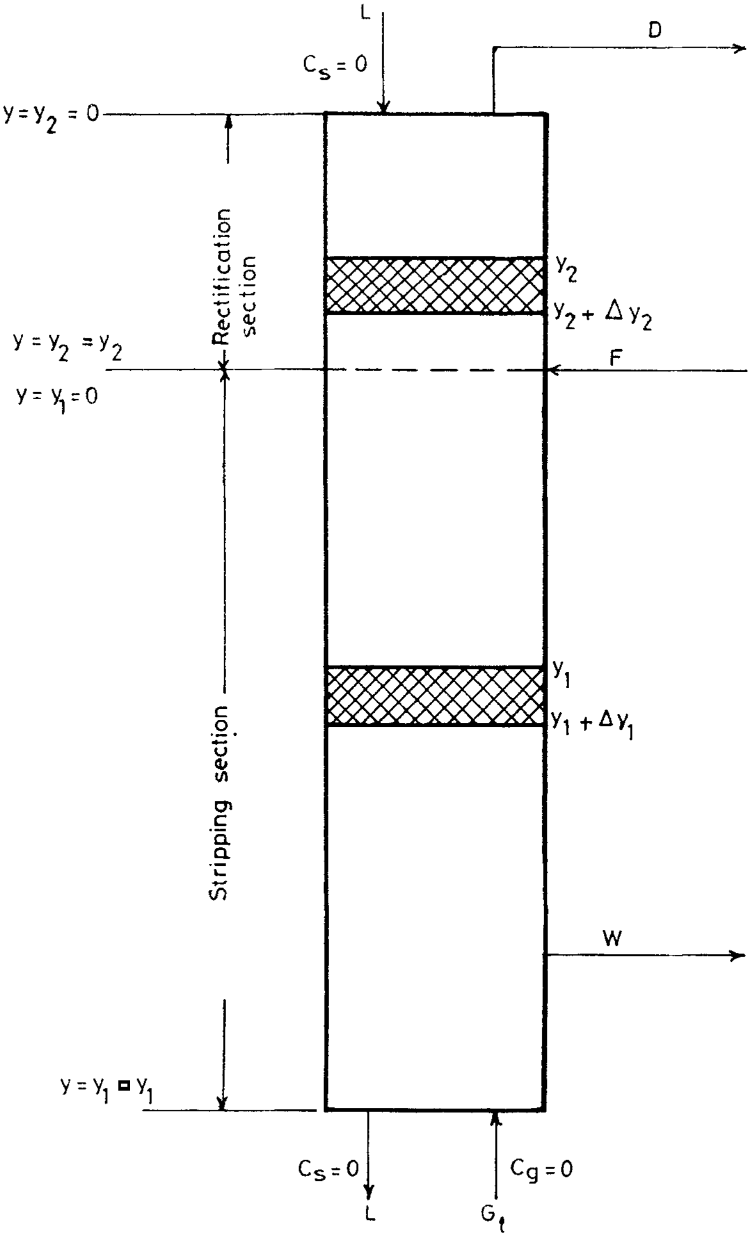

FIG. 2 Schematic diagram of boundary conditions for the chromatographic column.

Mass balance for the fluid phase:

$$(U_gAeC) - \left[U_gAeC - \frac{\partial(U_gAeC)}{dy}dy\right] = \left[\frac{\partial(eACdy)}{dt}\right] + ka(C - C_1)Ady$$

or

$$e\frac{\partial C}{\partial t} - eU_g\frac{\partial C}{\partial y} = ka(C_1 - C) \tag{2}$$

Mass balance for the solid phase:

$$(1 - e)U_s\frac{\partial C_s}{\partial y} + (1 - e)\frac{\partial C_s}{\partial t} = ka(C - C_1) \tag{3}$$

Equations (2) and (3) are the basic equations used to formulate the mathematical model. The system can be simplified by dividing the column into two sections: above the feed (rectification section) and below the feed (stripping section). Hence, two mathematical expressions should be obtained, dependent on the boundary conditions.

For the Stripping Section

It can be seen from Fig. 2 that the initial and boundary conditions which are valid:

I.C. at $t = 0$: $C = C_s = 0$
B.C.1 at $y = y_1 = 0$: $C = f(t),\ C_s = g(t)$
B.C.2 at $y = y_1$: $C = C_s = 0$

where $f(t)$ and $g(t)$ are expressions of C and C_s as functions of time respectively. Equations (2) and (3) are changed to ordinary differential equations by using the following dimensionless variables:

$$\eta_1 = \frac{y}{y_1}; \qquad \tau = \frac{U_g}{U}(tU_s/y_1 - \eta) \tag{4}$$

By using the above dimensionless expressions and applying the change of variables technique, Eqs. (2) and (3) can be rewritten as:

$$\text{For fluid phase: } \frac{\partial C}{\partial \tau} - \frac{\partial C}{\partial \eta} = \psi_g(C_s - C) \tag{5}$$

$$\text{For solid phase: } \frac{\partial C_3}{\partial \eta} = \psi_s(C - C_s) \tag{6}$$

where $\psi_g = aky_1/eU_g$; $\psi_s = aky_1/(1 - e)U_s$

Rewrite the initial and boundary conditions in dimensionless form:

$$\left.\begin{array}{llll} \text{I.C.} & \text{at} & \tau = 0: & C = C_s = 0 \\ \text{B.C.1} & \text{at} & \eta = 0: & C = f(\tau),\ C_s = g(\tau) \\ \text{B.C.2} & \text{at} & \eta = 1: & C = C_s = 0 \end{array}\right] \tag{7}$$

Abid (20) found an analytical expression of $f(\tau)$ using the experimental data of Aadhami (21) for separating benzene from cyclohexane in the case of laboratory moving-bed chromatography.

The generalized expression stated in Eq. (8) is successful for all cases involving continuous column chromatography:

$$C = C_{-}[1 - \exp(\lambda\tau)] \tag{8}$$

Equations (5) and (6) can be solved by Laplace transformation for the concentration distribution of a fluid component at any time along the stripping section of the column. When the experimental data of Yoon and Kunii (22) and Aadhami (21) are used, insignificant terms that would have a minor effect on C are ignored. The resulted equation can be simplified to the form:

$$C(\eta, \tau) = \frac{C_{-}[1 - \exp(\psi_s - \psi_g)(1 - \eta_1)]}{[1 - \exp(\psi_s - \psi_g)]} \tag{9}$$

$$+ C_{-}\lambda \sum_{n=1}^{\infty} \left[\frac{(H \exp A_1 - E \exp A_2) \exp D_1}{D_2} + \frac{(Q \exp A_1 - N \exp A_2)}{D_3} \right]$$

For the Rectification Section

Equations (2) and (3) or the dimensionless equations (5) and (6) can be used to obtain the equation of concentration profile within the rectification section by using the boundary conditions

$$\begin{array}{llll} \text{I.C.} & \text{at} & t = 0: & C = C_s = 0 \\ \text{B.C.1} & \text{at} & y_2 = 0: & C = f(t),\ C_s = 0 \\ \text{B.C.2} & \text{at} & y_2 = y_2: & C = f_1(t),\ C_s = g_1(t) \end{array}$$

or in dimensionless form

$$\begin{array}{llll} \text{I.C.} & \text{at} & \tau = 0: & C = C_s = 0 \\ \text{B.C.1} & \text{at} & \eta_2 = 0: & C = f(\tau),\ C_s = 0 \\ \text{B.C.2} & \text{at} & \eta_2 = 1: & C = f_1(\tau),\ C_s = g_1(\tau) \end{array}$$

where $\eta_2 = y/y_2$.

The fluid composition at the top and bottom points of the rectification section varies with time in the same manner as that found for the stripping section, and hence a similar analytical formula for $f(\tau)$ and $f_1(\tau)$ can be expressed.

$$C = f(\tau) = S_1[1 - \exp(\omega\tau)] \tag{10}$$

$$C = f_1(\tau) = S_2[1 - \exp(\omega_1\tau)] \tag{11}$$

where S_1, S_2, ω, and ω_1 are constants. They can be evaluated as before by using the conditions of the rectification section.

The differential equations for the rectification section are solved by using the Laplace transformation in the same procedure as that employed for the stripping section, and hence the following expression of C is obtained.

$$\begin{aligned} C(\eta, \tau) = {} & S_1[1 - \exp(\xi + \sigma + \omega\tau) + (S_2 - S_1)] \frac{1 - \exp(\psi_g - \psi_s)\eta_2}{1 - \exp(\psi_g - \psi_s)} \\ & + \left. \frac{S_1[\exp(\xi + \sigma)\eta_2 - \exp(\xi - \sigma)\eta_2] \exp z\tau}{2\sinh(\sigma)} \right|_{z=\omega} \\ & - \left. \frac{S_2[\exp(\xi + \sigma)\eta_2 - \exp(\xi - \sigma)\eta_2] \exp(\xi + z\tau)}{2\sinh(\sigma)} \right|_{z=\omega_1} \end{aligned} \tag{12}$$

Equation (12) can be used to predict the concentration distribution of any component in the fluid at a given time along any point in the rectification section of the column.

Dimensionless Concentration Distribution

The concentration distributions in both sections of the column are related on the same basis by defining a dimensionless form of concentration, i.e., by dividing Eqs. (9) and (12) by the highest concentration of the light component at the steady-state condition within the column. The highest concentration has been found to be located just above the feed point. The dimensionless concentrations can be taken as (C^{*1}) and (C^{*2}) for the stripping and rectification sections, respectively.

RESULTS AND DISCUSSION

To illustrate the accuracy of the present model in predicting the concentration distribution along the separating column, the operating conditions of Yoon and Kunii (22) and Aadhami (21) were used as input information to the mathematical model. The model is then used to calculate the concentration distribution along the column at any period of time measured from start up. Figures 3 and 4 compare the application of the present

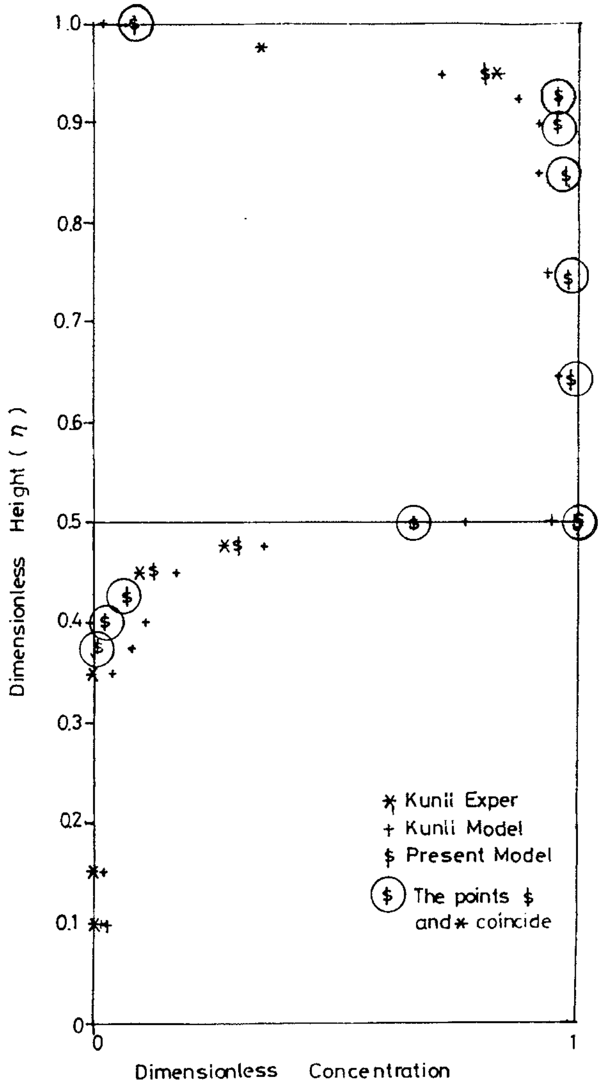

FIG. 3 Dimensionless concentration distribution (for ethane–air system) based on steady-state mass transfer coefficient.

model with that of the Yoon and Kunii model (22) and with the experimental results for ethane–air and ethylene–air systems, respectively, while Table 1 gives the average percentage deviation of the two models from the experimental results for the steady-state condition.

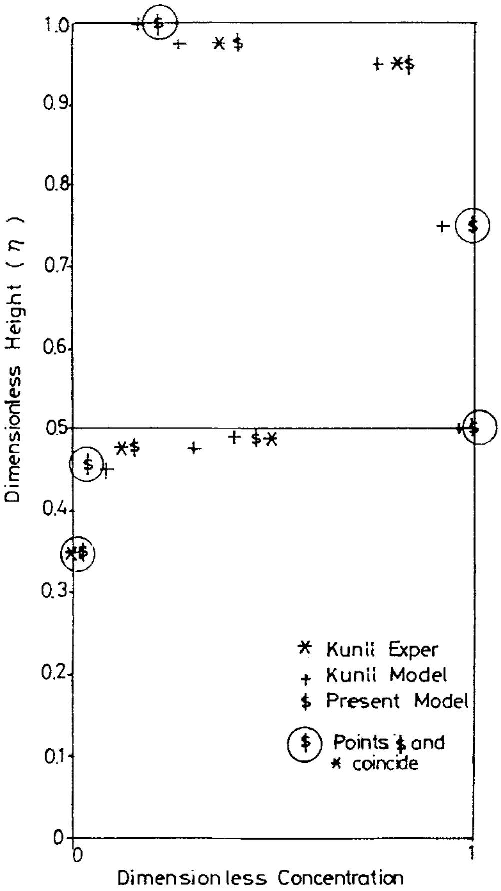

FIG. 4 Dimensionless concentration distribution (for ethylene–air system) based on steady-state mass transfer coefficient.

TABLE 1
Percent Deviation of Models from Experimental Results

System	Yoon and Kunii model	Present model
Ethane–air	11.09	2.10
Ethylene–air	10.62	2.31

The accuracy of the present model for prediction of the experimental data during the unsteady-state period was tested by using the operating data of Aadhami (21). The separation of cyclohexane from benzene was used as input data to the mathematical model. The predicted concentrations as functions of the column height are compared graphically in Fig. 5. The model predicts values within 17.8, 14.1, and 7.2% deviation from the experimental data for 5, 25, and 85 minutes from start-up, respectively.

An attempt was made to improve the applicability of the present model by finding the variation of the mass transfer coefficient with time as shown

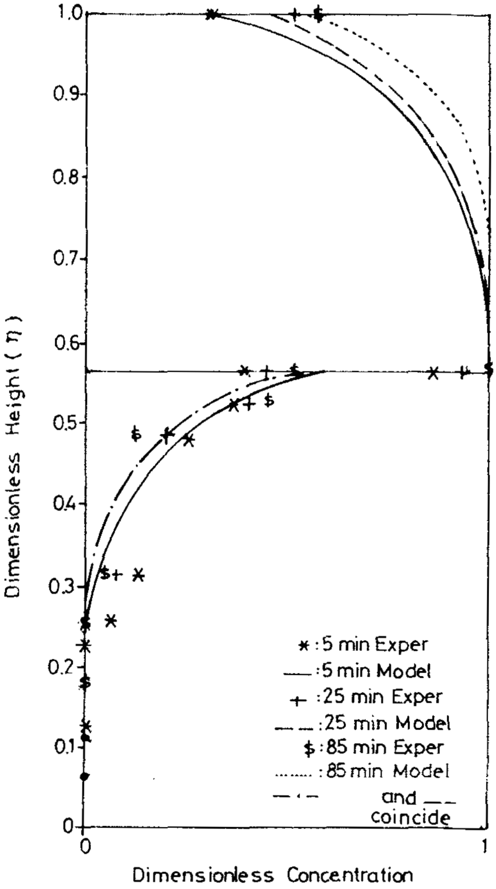

FIG. 5 Dimensionless concentration distribution (for cyclohexane–benzene system) based on steady-state mass transfer coefficient.

in the Appendix. An extensive study was made to find the behavior of the mass transfer coefficient during the unsteady-state period of separation.

The mathematical correlation for instantaneous mass transfer was obtained by using the least-squares method for curve fitting of the experiment data:

$$k' = k[1 - \exp(\phi\tau)] \tag{13}$$

where k' and k (in cm/h) are the instantaneous and steady-state gas mass transfer coefficient, respectively. k' is now used instead of k in Eqs. (9) and (12) to recalculate the dimensionless concentration distribution. The predicted results are represented graphically in Fig. 6. The average percentage deviations of the predicted data as compared with the experimental data are reduced to 7.06, 6.3, and 6.15 for periods of 5, 25, and 85 minutes after start-up, respectively. Therefore, using Eq. (13) has increased the applicability of the model for finding the concentration distribution during instantaneous periods of separation.

The derived model can also be used to investigate how the various operating parameters affect the performance of the continuous moving-bed chromatographic column. Figures 7 and 8 represent the variation of feed composition with the bottom reflux ratio when the top and bottom products are kept constant. Both figures are plotted using the operating conditions of Aadhami (21). The figures indicate that the composition of the bottom product is more sensitive to the reflux ratio variation than is the top product composition.

Design Procedure

The following steps are recommended as a design procedure for a continuous moving-bed chromatographic column in the case of dilute mixtures at steady-state conditions.

Mass Balance

The column can be divided into three sections as shown in Fig. 9. The total mass flow rate of the feed (g/h) is

$$F = F_l + F_h \tag{14}$$

$$y_l = \frac{F_l}{F}, \qquad y_h = \frac{F_h}{F}$$

The overall mass balance around the separating column is

$$F = D + W \tag{15}$$

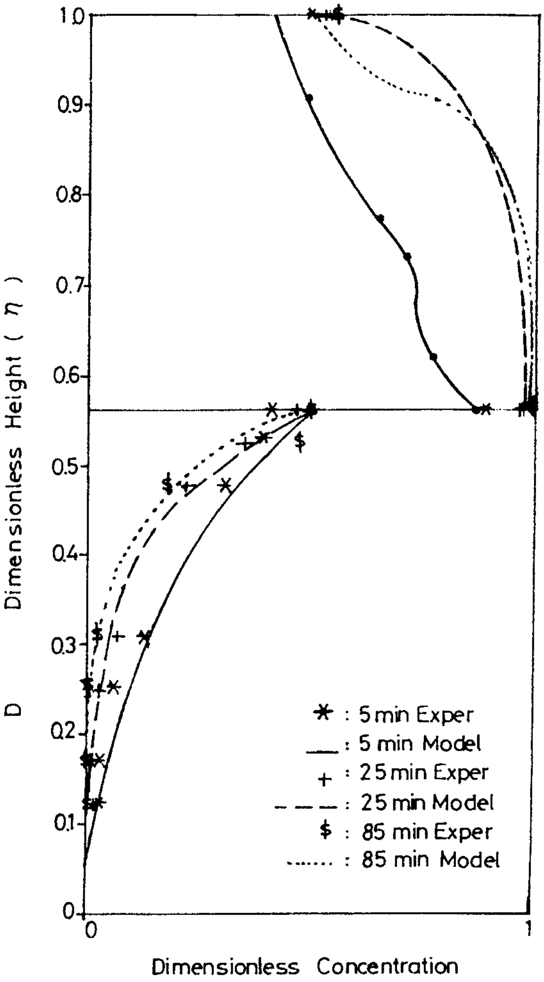

FIG. 6 Dimensionless concentration distribution (for cyclohexane–benzene system) based on instantaneous mass transfer coefficient.

The material balance for the light component is

$$Fy_l = Dy_{lD} + Wy_{lw} \tag{16}$$

From Eqs. (15) and (16):

$$D = \left(\frac{y_{lD} + y_{lw}}{y_l - y_{lw}}\right) F \tag{17}$$

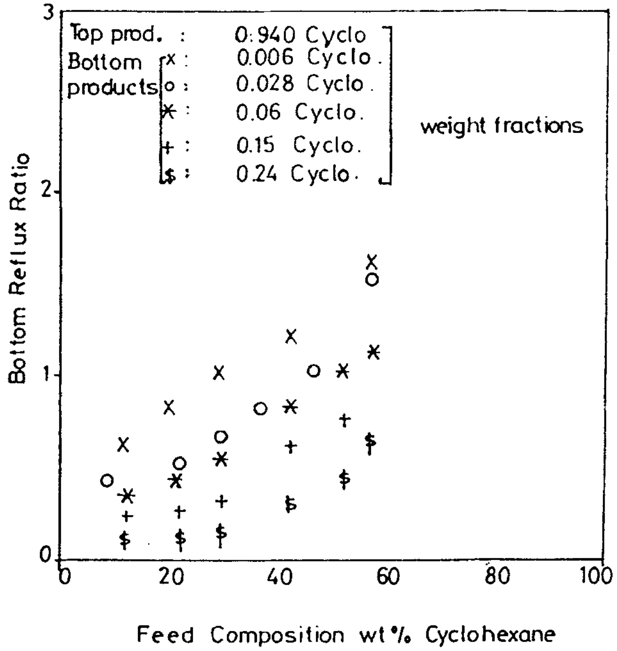

FIG. 7 Effect of feed composition on bottom reflux ratio at constant top product composition.

Define R as the bottom reflux:

$$R = G|G_w = G|(G_t - G_s) \tag{18}$$

or

$$G = G_t R|(1 + R) \tag{19}$$

If the volumetric flow rates of heavy component in stream W and at point M are G_{hw} and G_{hM}, respectively, then

$$G_{hw} = V_{hw} W \tag{20}$$

$$G_{hM} = V_{hw} WR \tag{21}$$

where V_{hw} is the retention volume of the heavy component in the bottom product.

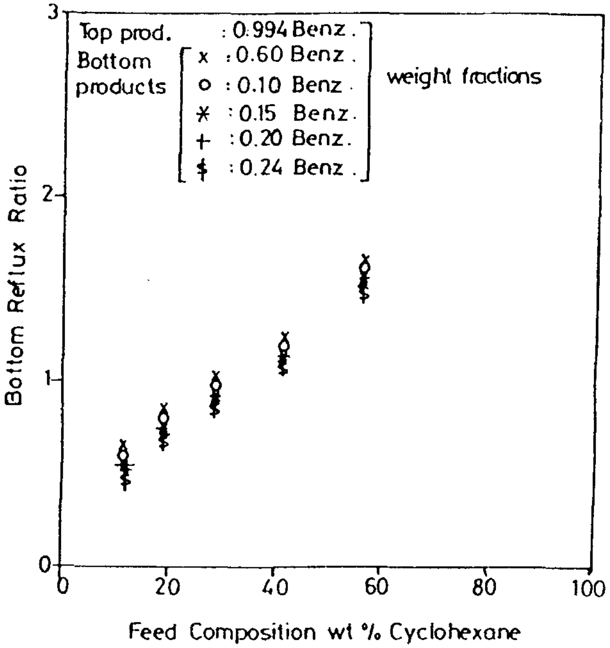

FIG. 8 Effect of feed composition on bottom reflux ratio at constant bottom product composition.

The volume of the heavy component in the bottom product is $V_{hw}W$. Since the density of the heavy component vapor just above the bottom product is equal to that of the bottom product, its quantity at point P is RW.

The concentration of the light component (L.C.) just above the bottom product is

$$C_{lw} = \frac{y_{lw}WR}{G}$$

while the concentration of L.C. at the top product is

$$C_{lD} = \frac{y_{lD}D}{G}$$

By examining the experimental data of Aadhami, it can be found at the steady-state condition that the amount of L.C. in the feed is equal to the

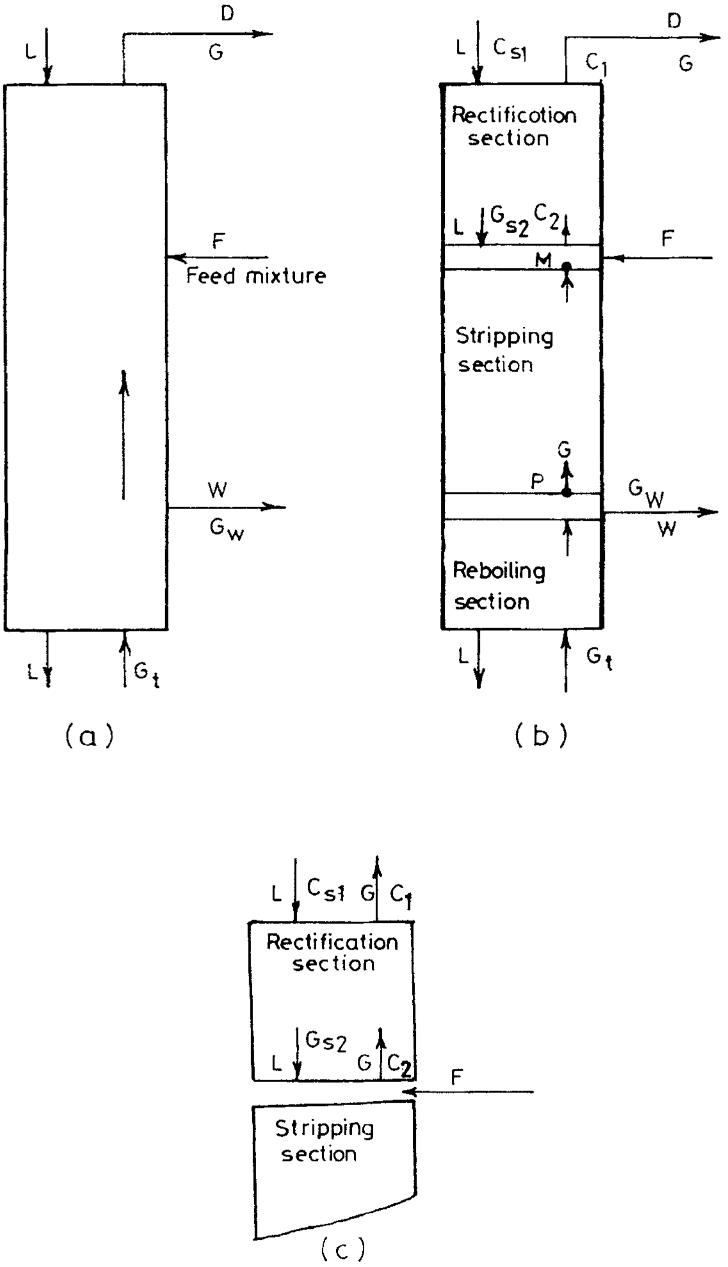

FIG. 9 Schematic diagrams of column sections to be used in mass balance.

amount of L.C. at a point just below the feed. Hence, if the concentration of L.C. just below the feed is

$$C_{1M} = \frac{y_1 F}{G} = y_{1M}F/G$$

the concentration of L.C. in the column just above the feed point is

$$C_2 = 2y_1F/G$$

Calculating the $(L/G)_{min}$

The $(L/G)_{\text{min}}$ ratio of any system depends on:

1. The nature of the adsorbent
2. The nature of the binary mixture
3. The type of isotherm used

The design procedure is recommended for a dilute mixture system which is usually represented by a linear isotherm. Hence, the Langmuir relation or Henry's law can be used:

$$C_s = kC^* \tag{22}$$

By setting a mass balance for L.C. around the rectification section:

$$GC_2 + LC_{s1} = GC_1 + LC_{s2} \tag{23}$$

Using the conditions at the top of the rectification section gives

$$C_1 = 0 \quad \text{and} \quad GC_{1D} = y_{LD}D$$

and hence Eq. (23) is reduced to

$$GC_2 = (y_{LD}D) + LC_{s2} \tag{24}$$

Sherwood [cited in Jenson and Jefferys (23)] assumed that a local equilibrium is established at the steady-state condition for any component between the solid and gas phases at any point throughout the column. By using this assumption, Eq. (22) can be rewritten as

$$C_{s2} = KC_2 \tag{25}$$

Substituting Eq. (25) into Eq. (24) and using

$$C_2 = 2y_1F/G$$

the following equation is obtained:

$$L/G = (2y_1F - y_{1D}D)/(2Ky_1F) \tag{26}$$

The assumption used in obtaining Eq. (25) makes Eq. (26) give the minimum value of L/G, hence

$$(L/G)_{\min} = (2y_1F - y_{1D}D)/(2Ky_1F) \tag{27}$$

or

$$(L)_{\min} = G(2y_1F - y_{1D}D)/(2Ky_1F) \tag{28}$$

since

$$L = R_m L_{\min} \tag{29}$$

where L is the optimum operating volumetric solid flow rate and R_m is a variable greater than 1 and can be determined by trial and error to give the best fit of the results.

Calculation of Longitudinal Pressure Drop

The longitudinal pressure drop along the column of moving-bed systems is well predicted by Yoon and Kunii when using Ergun's equation:

$$(\Delta p/l) = 150\mu U(1 - e)^2/d^2e^3 + 175U^2\rho(1 - e)/de^3 \tag{30}$$

where

$$\begin{aligned} U &= \text{fluid velocity with respect to a moving bed} \\ &= U_s + U_g \end{aligned} \tag{31}$$

$$U_g = G/eA \tag{32}$$

$$U_s = 1/(1 - e)A \tag{33}$$

μ and ρ are the viscosity and density of the fluid, respectively, and d_1 is the solid particle diameter.

Estimation of Column Dimensions

The dimensions of the column can be calculated by the iteration method. The column dimensions are initially assumed, and then by using Eqs. (9) and (12) the concentration distribution along the column is calculated. The optimum design dimensions are reached when the calculated concentrations of certain components at the top and the bottom of the column agree with the required values.

CONCLUSION

Equations are derived to predict the concentration distribution along a moving-bed chromatographic column. The equations give a good representation of experimental data during the unsteady- and steady-state working periods when instantaneous mass transfer is used.

The design procedure is recommended for use in designing new adsorbers or for improving the efficiency of existing ones.

APPENDIX

Behavior of Mass Transfer Coefficient

Extensive work was carried out to investigate the behavior of the mass transfer coefficient during the unsteady-state period of adsorption. The fundamental equations can be used to compute the height of the packed tower, as shown by Perry and Chilton (24).

$$Y = N_{OG}H_{OG} \tag{1A}$$

where

$$H_{OG} = G_m/ka \tag{2A}$$

$$N_{OG} = \int_{C_1}^{C_2} dC/(C - C^*) \tag{3A}$$

Hence

$$(ka) = N_{OG}G_m/Y \tag{4A}$$

Treyball (25) suggested use of the logarithmic mean driving force for the linear isotherm, and hence

$$N_{DG} = (C_2 - C_1)\ln[(C - C^*)_1/(C - C^*)_2]/[(C - C^*)_1 - (C - C^*)_2] \tag{5A}$$

where $(C - C^*)_1$ and $(C - C^*)_2$ are the driving force at the top and bottom of the column, respectively. By using the equations of operating lines at each section, Eq. (5A) is rearranged to give, for the stripping section:

$$N_{OG} = \ln\left[\frac{M_s/LK - C_s(1 - G/LK)}{M_s/LK - C_-(1 - G/LK)}\right] \Big/ (1 - G/LK) \tag{6A}$$

and for the rectification section:

$$N_{OG} = \ln\left[\frac{M_D/LK - C_2(1 - G/LK)}{M_D/LK - C_1(1 - G/LK)}\right] \Big/ (G/LK - 1) \tag{7A}$$

where M_s and M_D are the mass flow rates of solute in grams per hour leaving with the bottom and top products, respectively.

The unsteady-state experimental data of Aadhami (21) are used to calculate the NTU for both sections, and consequently the mass transfer coefficient is found to increase exponentially with time during the transient period:

$$k' = k[1 - \exp(\phi\tau)]$$

where ϕ is a constant evaluated from the operating conditions and geometry of the column.

NOMENCLATURE

a	specific surface area of bed (cm^2/cm^3)
A	cross-sectional area of the column (cm^2)
A_1	$= -U_n + (\psi_g - \psi_s + 3U_n)\eta_1/2$
A_2	$= U_n + (\psi_g - \psi_s - U_n)\eta_1/2$
C	concentration of solute in the fluid through the column (g solute/mL carrier gas)
C^*	equilibrium concentration of solute (g solute/mL carrier gas)
C^{*1}	dimensionless concentration for stripping section
C^{*2}	dimensionless concentration for rectification section
C_1	concentration of solute in the top product (g/mL carrier gas)
C_2	concentration of solute just above the feed point (g solute/mL carrier gas)
C_-	steady-state concentration of solute just below the feed point (g solute/mL carrier gas)
C_1	concentration of solute at the interface (g solute/mL solid)
C_s	concentration of solute on solid (g solute/mL solid)
C_w	concentration of solute in the bottom product (g solute/mL carrier gas)
d	solid particle diameter (cm)
D	mass flow rate of top product (g/h)
D_1	$= -(\psi_s + \psi_g)Y$
D_2	$= (2V_n^2 U_n)^2 + (U_n^2 V_n - V_n^3)^2$
D_3	$= (2V_n^2 U_n)^2 + (V_n^3 - U_n^2 V_n)^2$
e	block porosity or fractional voidage of bed particles or packing
F	mass flow rate of feed (g/h)
F_l	mass flow rate of light component in the feed (g/h)
F_h	mass flow rate of heavy component in the feed (g/h)
G	volumetric flow rate of gas carrier at point p (mL/h)
G_{hM}	volumetric flow rate of heavy component at point M (mL/h)

G_{hw}	volumetric flow rate of heavy component in the bottom product (mL/h)
G_M	volumetric flow rate of gas carrier at point M (mL/h)
G_t	total volumetric flow rate of gas carrier (mL/h)
G_w	volumetric flow rate of gas carrier in the bottom product (mL/h)
H	$= [2V_n^2U_n \cos(3V_n/2 + n\pi)y - V_n - n\pi + \tau V_n) - U_n^2V_n - V_n^3) \sin[(3V_n/2 + n\pi)y - V_n - n\pi + \tau V_n)]$
H_{OG}	the height of the overall transfer units (cm)
k	steady-state gas mass transfer coefficient [g/h·cm^2(g/mL)]
$k^{\cdot}$	instantaneous gas mass transfer coefficient [g/h·cm^2(g/mL)]
K	partition coefficient of solute [(g solute/mL solid)/(g solute/mL carrier gas)]
l	length of bed or height of column packing (cm)
L	volumetric flow rate of solid (mL/h) or of liquid (cm^3/h)
M_s	mass flow rate of solute leaving with the bottom product (g/h)
M_D	mass flow rate of solute leaving with the top product (g/h)
n	integer, ranges from $-\infty$ to $+\infty$
N	$= [2V_nU_n \cos((V_n/2 - n\pi)y - V_n\pi - V_n + n\pi) - (V_n^3 - U_n^2V_n) \sin((V_n/2 - n\pi)y - V_n\pi - V_n + n\pi)]$
N_{OG}	number of transfer units
p	variable and assumed to be real Laplace transform of $f(x) = \bar{f}(p)$
Q	$= [2V_nU_n \cos(V_n - n\pi - \pi V_n - (3V_n/2 - n\pi)\eta_1) - (V_n^3 - U_n^2V_n) \sin(V_n - n\pi - \pi V_n - (3V_n/2 - n\pi)\eta_1)]$
R	reflux ratio
R_m	minimum reflux ratio
S_1	constant in Eq. (10)
S_2	constant in Eq. (11)
t	time (s)
U	fluid velocity with respect to moving bed (cm/s)
U_g	fluid velocity with respect to stationary vessel (cm/s)
U_s	solid velocity (cm/s)
V_{hw}	the retention volume of the heavy component in the bottom product (cm^3)
W	mass flow rate of solute with the bottom product (g/h)
y	distance measured from the top of the test section (cm)
y_1	height of stripping section (cm)
y_2	height of rectification section (cm)
y_h	F_h/F
y_l	F_l/F
y_{lD}	the fraction of the light component of solute in the top product

y_{lw}	the fraction of the light component of solute in the bottom product
z	ranges from zero to 20

Greek Letters

$\Delta p/l$	longitudinal pressure drop (g/cm^2·cm)
ζ	$= (p + \psi_g + \psi)/2$
η_1	dimensionless distance y/y_1
η_2	dimensionless distance y/y_2
λ^1	constant in Eq. (8)
μ	carrier fluid viscosity (g/cm·s)
ρ	carrier fluid density (g/mL)
σ	$= [(p + \psi_g + \psi_s)^2/4) - \psi_g\psi_s]^{0.5}$
τ	dimensionless time
ψ_g	aky_1/eU_g
ψ_s	$aky_1/(1 - e)U_s$

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